Assessing an LPG engine performance through mathematical modelling and simulation

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The objective of this study was to use a mathematical model to carry out the cycle analysis of a four-stroke, four-cylinder gasoline engine equipped with LPG system and to try to approximate the true engine power by using the postulated model. Temperature, pressure, work, and power were determined through simulation for the maximum torque rating of 3000 rpm. The values of experimental engine power and the calculated power were 36.3 kW, with about 1 % error. Power was calculated at 2000, 2500 and 5500 rpm to assess the usefulness of the model in estimation of the true engine power, too. Error varied between 2 to 3.7 % at these engine speeds, implying the need for further development in the combustion model and in the simulation code for more accurate prediction of the true engine power.

K e y w o r d s: LPG, cycle analysis, mathematical model, simulation

Nomenclature

$(ash)_\xi$	– mole ratio of ash in fuel containing 1 kg carbon [kmol ash/kmol C]
A_{ht}	– area of heat transmission [m^2]
C	– carbon ratio in field [kg C/kg fuel]
c_{f}	– carbon mole ratio in 1 kmol fuel [kmol C/kmol fuel]
C_{p}	– carbon ratio per 1 kmol fuel [$\mathrm{kJ\ kg^{-1}\ K^{-1}}$]
C_{V}	– specific heat at constant volume [$\mathrm{kJ\ kg^{-1}\ K^{-1}}$]
C_α	– C mole ratio in fuel containing 1 kg carbon [kmol C/kmol C]
CO_2	– $\mathrm{CO_2}$ mole ratio in fuel containing 1 kg C [kmol $\mathrm{CO_2}$/kmol C]
d_{nf}	– fuel burned per crankshaft revolution [kg]
E	– internal energy [kJ]
e_{i}	– specific internal energy of the mixture [$\mathrm{kJ\,kg^{-1}}$]
H	– heat transfer coefficient [$\mathrm{kJ\,m^{-2}\,h^{-1}\,K^{-1}}$]
H_{f}	– hydrogen ratio in fuel [kg H/kg fuel]
h_{i}	– specific enthalpy [$\mathrm{kJ\,kg^{-1}}$]
H_{p}	– hydrogen ratio per 1 kmol fuel [kmol $\mathrm{H_2}$/kmol fuel]

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H_β – H mole ratio in fuel containing 1 kg carbon [kmol H/kmol C]
H_2O – H_2O mole ratio in fuel containing 1 kg C [kmol H_2O/kmol C]
$(H_2O)_\xi$ – H_2O mole ratio in fuel containing 1 kg carbon [kmol H_2O/kmol C]
k – polytrophic coefficient [–]
m_f – amount of fuel [kg]
n – number of moles [number]
N_2 – nitrogen ratio in atmosphere [%]
N_δ – N mole ratio in fuel containing 1 kg carbon [kmol N/kmol C]
n_DS – number of revolutions per second [s^{-1}]
$n_{\mathrm{H_2O}}$ – mole number of water in air [kmol]
$n_{\mathrm{N_2}}$ – mole number of nitrogen in air [kmol]
$n_{\mathrm{O_2}}$ – mole number of oxygen in air [kmol]
n_da – mole number of dry air [kmol]
$n_{\mathrm{da_{min}}}$ – mole number of minimum amount of dry air required for combustion [kmol]
n_ex – mole number of exhaust gas [kmol]
n_f – mole number of fuel filled into cylinder [kmol]
n_fill – mole number of inlet air and fuel [kmol]
n_ma – mole number of moist air [kmol]
$n_{\mathrm{ma_{min}}}$ – mole number of minimum amount of moist air required for combustion [kmol]
n_mix – mole number of overall mixture in cylinder [kmol]
O – oxygen in 1 kmol fuel [kmol O/kmol fuel]
O_2 – O_2 mole ratio in fuel containing 1 kg C [kmol O_2/kmol C]
O_min – sufficient O_2 required for combustion [kmol O_2/kmol fuel]
O_γ – O mole ratio in fuel containing 1 kg carbon [kmol O/kmol C]
$P_{1,2}$ – pressures at successive crankshaft angles [$\mathrm{N\,m^{-2}}$]
$P_{\mathrm{M_{abs}}}$ – percentage of absolute moisture [%]
$P_{\mathrm{H_2O}}$ – percentage of water in air [%]
$P_{\mathrm{N_2}}$ – percentage of hydrogen in air [%]
$P_{\mathrm{O_2}}$ – percentage of oxygen in air [%]
P_atm – atmospheric pressure [$\mathrm{N\,m^{-2}}$]
P_db – percentage of dry based water vapour in air [$\mathrm{N\,m^{-2}}$]
P_ex – exhaust pressure [$\mathrm{N\,m^{-2}}$]
P_exc – exhaust closing pressure [$\mathrm{N\,m^{-2}}$]
P_exo – exhaust opening pressure [$\mathrm{N\,m^{-2}}$]
P_fill – pressure of inlet air and fuel [$\mathrm{N\,m^{-2}}$]
P_in – intake air pressure [$\mathrm{N\,m^{-2}}$]
P_mix – mixture pressure [$\mathrm{N\,m^{-2}}$]
P_sat – saturation pressure [$\mathrm{N\,m^{-2}}$]
P_wb – percentage of wet based water vapour in air [%]
RH – relative humidity [%]
Q – heat transferred to cooling water [kJ]
Q_rh – reaction heat of fuel [kJ]
R – universal gas constant [kJ/kmol K]
S – sulphur ratio in fuel [kg $\mathrm{kmol^{-1}}$]
S_p – S ratio in 1 kmol fuel [kmol S/kmol Y]
S_ε – S mole ratio in fuel containing 1 kg carbon [kmol S/kmol C]
$T_{1,2}$ – temperatures at successive crankshaft angles [K]
T_atm – temperature at atmospheric conditions (293 K) [K]
T_ex – exhaust gas temperature [K]
T_exc – exhaust closing temperature [K]
T_exo – exhaust opening temperature [K]
T_ext – cylinder external temperature [K]
T_fill – temperature of inlet air and fuel [K]
T_int – cylinder internal temperature [K]
T_mix – mixture temperature [K]
$V_{1,2}$ – volumes at successive crankshaft angles [$\mathrm{m^3}$]
V_c – volume of cylinder [$\mathrm{m^3}$]
V_exc – exhaust gas closing volume [$\mathrm{m^3}$]
V_fill – sum of volumes of intake air and fuel [$\mathrm{m^3}$]
V_ma – moist air volume [$\mathrm{m^3}$]
V_mix – mixture volume [$\mathrm{m^3}$]
V_vf – fuel volume in vapour phase [$\mathrm{m^3}$]
W – work [kJ]
W_da – molecular weight of dry air [kg $\mathrm{kmol^{-1}}$]

W_f – molecular weight of fuel [kg kmol^{-1}]
W_ma – molecular weight of moist air [kg kmol^{-1}]
α – molecular weight ratio of carbon [%]
β – molecular weight ratio of hydrogen [%]
E – molecular weight ratio of sulphur [%]
Λ – excess air coefficient [–]
ρ_at – density of moist air [kg m^{-3}]
σ – minimum oxygen required for theoretical complete burning of fuel [kmol O_2/kmol C]
η – N mole ratio in combustion air [kmol N/kmol C]
θ – crankshaft angle [°]
θ_it – ignition timing [°]
ΔT – temperature increase of fresh mixture due to temperatures of exhaust gases in the cylinder [K]
$\Delta\theta$ – combustion interval [°]

1. Introduction

The gaseous emissions from vehicles have been a major concern worldwide in recent decades due to increased air pollution. Much of emissions are known to result from gasoline and diesel engines. LPG is cleaner than petrol and diesel since it is predominantly composed of simple hydrocarbon compounds. Compared with emissions from vehicles on petrol and diesel, emissions from LPG-driven vehicles contain lower levels of hydrocarbon compounds, nitrogen oxides, sulphur oxides, air toxics, and particulates [6]. In many countries, there has been increasing public awareness, requiring auto industry to seek venues to decrease emissions. The governments are encouraged to establish incentive policies for cleaner vehicle production [13] and most have already established clean air acts to meet the demand for cleaner and cheaper fuels. Another forcing factor to develop better vehicle emission technologies was the cost of traditional fuels. As a result, alternative fuels such as compressed natural gas (CNG) and liquefied petroleum gas (LPG) have been introduced. Due to cost effectiveness of LPG, consumers tend to install LPG systems aiming at not only less fuel costs but reduced emissions [1, 4, 10]. Millions of vehicles are equipped with LPG systems in the world. South Korea leads in LPG use followed by others such as Japan, Turkey, Egypt, Australia, and Italy.

Much experimental research has been conducted to assess the emissions from engines. These studies also include emissions from dual fuel (gasoline + LPG) engines. Research reports on power measurement of engines while running on LPG can also be found in the literature. Such experimental studies are invaluable in generating information on how to reduce emissions and determine the most proper operating conditions of engines. Other efforts are devoted to developing mathematical models to estimate the performance of LPG engines at different engine speeds and settings. These theoretical models are expected to simulate experimental results in terms of different parameters such as cylinder temperature, pressure, and engine power at different engine speeds and specified settings. Therefore engineering calculations relevant to cycle analysis of an internal combustion engine need to be used to develop theoretical tools since these models become handy to simulate the attributes that are of interest in the analysis of engine performance.

The objective of this study was to postulate a mathematical model to do cycle analysis of an LPG engine and simulate the model to estimate the power of the engine to verify the applicability of the model and computer code that was developed.

2. Materials

2.1. Motor parameters

A four-cylinder, four-stroke-cycle, gasoline engine (Kartal 1.4) that was equipped with an LPG system was used in the study. The technical properties of the engine are shown in Table 1. The engine tests were conducted by the manufacturer (Fiat-Tofaş) using the LPG system mounted on the engine, which provided power at different engine speeds along with some other data. The experimental engine power at different engine speeds was used to estimate the true engine power provided by the manufacturer in this study.

Table 1. Properties of reference gasoline engine

Motor property	Dimension
Cylinder diameter	86.4 mm
Number of cylinders	4
Stroke	67.4 mm
Total stroke volume	1581 cm^3
Compression ratio	9.2 : 1
Rod length	172.8 mm

2.2. Properties of LPG

LPG produced by Kirikkale Refinery in Turkey was used in this study. Technical properties of LPG depend on the properties of cruel oil to be processed. Therefore, averages resulting from elemental analyses that were conducted by the producer over one year were calculated for this study (Table 2). While LPG consists mostly of propane (C_3H_{10}) and butane (C_4H_{10}), there are propylene and butylenes in LPG as well.

Table 2. LPG constituents as obtained from gas chromatography

Gas	Chemical formulation	Volume ratio in LPG [%]	Gas	Chemical formulation	Volume ratio in LPG [%]
Methane	CH_4	0.40	n-butane	C_4H_{10}	22.82
Ethane	C_2H_6	10.18	Butylenes	C_4H_{10}	0.19
Propane	C_3H_8	38.58	i-pentane	C_5H_{12}	0.55
Propylene	C_3H_8	0.05	n-pentane	C_5H_{12}	0.12
i-butane	C_4H_{10}	27.11			

3. Model development

Due to the complexities of combustion process, the governing equations for combustion were simplified and some assumptions were made during the development of the model. It was assumed that theoretical amount of air in the reaction zone that separates burned and unburned mixture zones was equal to the theoretical amount of air of the mixture. Based on excess air ratio, the calculations regarding specific heat could be done for stoichiometric mixture in the reaction zones or for the lean-burn mixture. The emphasis of this study will be on developing a simplified and applicable mathematical model that can describe the chemical reaction of the mixture in the cylinder so that the working cycle could be analysed with sufficient accuracy. Compression ratio, intake air pressure, intake air temperature, and exhaust gas pressure were assumed to be constants.

3.1. Cycle analysis

The cycle was completed in 5 steps: inlet, compression without combustion, compression with combustion, expansion with combustion, and expansion without combustion. Pressure, temperature, and work were calculated for each crankshaft angle. Calculations were done for each crankshaft revolution using the first law of thermodynamics. The values given in Janaf tables were used to do cycle analysis [2]. Gas volume varies linearly with the piston displacement. The change in gas volume was calculated per crank angle since piston displacement is not linearly related to crankshaft angle [9]. Work (W), heat transmitted to the cooling water (Q), and internal energy (E) were used in the second step of the cycle analysis (after intake stroke). Since internal energy is a function of temperature, temperature was determined using the first law of thermodynamics. In the third and fourth steps, compression and expansion with combustion were considered, respectively. Reaction heat of fuel (Q_{rh}) was added to the equation so that an energy equation could be obtained, in which heat transmitted to the cooling water, work, internal energy, and reaction heat were considered [7]. Expansion without combustion was included up to a crank angle of 540 degrees assuming no significant effect would occur in calculations for succeeding degrees of crankshaft angle.

3.2. Mathematical relations

The combustion process involves chemical processes combining oxygen from the air and carbon and hydrogen in the fuel. The final objective is to generate power during the expansion (power) stroke as heat is liberated while pressure increases during the process [5]. Thus, developing a combustion model requires equations relevant to fresh air and fuel that are filled into the cylinder along with some exhaust gas dragged back into the cylinder following the power stroke. Equations related to air components, number of moles for the air, thermo-chemical equations of the fuel, number of moles for fuel, and amount of fuel were determined before setting equations up that describe intake stroke, compression, and expansion.

3.2.1. Equations related to air and fuel

Thermo-chemical properties of the intake mixture were determined by using the equations related to air, fuel, and leftover gases in the cylinder [11, 3]. It was assumed that the air filled in the cylinder consists of 20.8 % oxygen and 79.2 % nitrogen by volume, relative humidity (RH) is 60 %, and saturation pressure (P_{sat}) is 2338 $\mathrm{N\,m^{-2}}$. The percentages of (wet based) water vapour in the air (P_{wb}), dry air (P_{db}), oxygen ($P_{\mathrm{O_2}}$), and nitrogen ($P_{\mathrm{N_2}}$) were calculated. Percentage of absolute moisture ($P_{\mathrm{M_{abs}}}$) was determined as a function of relative humidity (RH) in Eq. (1):

$$P_{\mathrm{M_{abs}}} = \frac{1}{1.608\,\{P_{\mathrm{atm}}/(P_{\mathrm{sat}}RH) - 1\}}. \tag{1}$$

Molecular weight of moist air (W_{ma}) was found using Eq. (2) and molecular weight of dry air (W_{da}) was taken to be 28.9644 $\mathrm{kg\,kmol^{-1}}$:

$$W_{\mathrm{ma}} = W_{\mathrm{da}}\frac{1 + P_{\mathrm{M_{abs}}}}{1 + 1.608(P_{\mathrm{M_{abs}}})}. \tag{2}$$

The final parameter related to air composition was the density of moist air (ρ_{at}) calculated as follows:

$$\rho_{\mathrm{at}} = \frac{P_{\mathrm{atm}}W_{\mathrm{ma}}}{RT_{\mathrm{atm}}}. \tag{3}$$

Thermo-chemical equations need to be set up to model the chemical reactions in the cylinder. Thus carbon (C_{p}), hydrogen (H_{p}), and sulphur percentages (S_{p}) in the fuel were first calculated. Then the molecular weight of fuel (W_{f}) was calculated using Eq. (4):

$$W_{\mathrm{f}} = 12.011C_{\mathrm{p}} + 1.008H_{\mathrm{p}} + 32.064S_{\mathrm{p}}. \tag{4}$$

The implicit formula of the fuel was expressed as $C_\alpha H_\beta O_\gamma N_\delta S_\varepsilon (H_2O)_\xi (ash)_\zeta$. Molecular weight ratios of C, S, and H were determined as follows for the fuel containing 1 kg of carbon:

$$\alpha = \frac{C_{\mathrm{p}}}{C_{\mathrm{p}}}, \tag{5}$$

$$\varepsilon = \frac{S_{\mathrm{p}}}{C_{\mathrm{p}}}, \tag{6}$$

$$\beta = \frac{H_{\mathrm{p}}}{C_{\mathrm{p}}}. \tag{7}$$

The amount of air required for combustion should be determined. The reaction formula, provided that 1 kmol of the fuel burns completely, was expressed as follows:

$$\begin{aligned} & C_\alpha H_\beta O_\gamma N_\delta S_\varepsilon (H_2O)_\xi (ash)_\zeta + \lambda(O_2 + 3.762N_2 + 7.656P_{\mathrm{M_{abs}}} H_2O)O_{\min} \\ & \quad \longrightarrow CO_2 + (0.5h + \xi + 7.656P_{\mathrm{M_{abs}}}\lambda O_{\min})H_2O + \varepsilon SO_2 + O_{\min}(\lambda - 1)O_2 \\ & \qquad + (0.5n + 3.762\lambda O_{\min})N_2. \end{aligned} \tag{8}$$

Sufficient or minimum oxygen required for combustion ($O_{\min}$) was calculated by using Eq. (9):

$$O_{\min} = (C + 0.25H + S + 0.5O). \tag{9}$$

Reaction formula can be expressed as follows when 1 kg kmol^{-1} carbon is found in the fuel and if all fuel burns during combustion:

$$\begin{aligned} & C_\alpha H_\beta O_\gamma N_\delta S_\varepsilon (H_2O)_\xi (ash)_\zeta + \lambda(O_2 + 3.762N_2 + 7.656P_{\mathrm{M_{abs}}} H_2O)\sigma \\ & \quad \longrightarrow O_2 + (0.5\beta + \xi + 7.656\lambda)H_2O + \varepsilon SO_2 + \sigma(\lambda - 1)O_2 \\ & \qquad + (0.5\delta + 3.762\lambda)N_2. \end{aligned} \tag{10}$$

Then complete combustion of the fuel can be expressed theoretically as follows:

$$\begin{aligned} & C_1 H_{2.125875} O_\gamma N_\delta S 0.000264694 (H_2O)_\xi (ash)_\zeta \\ & \quad + 1.0(O_2 + 3.762N_2 + 7.656 \times 0.0087067 H_2O)10.69072 \\ & \quad \longrightarrow \; CO_2 + (0.5 \times 2.12587 + 0.000264694 + 7.656 \times 1.0)H_2O. \end{aligned} \tag{11}$$

Minimum amount of oxygen in the case of theoretical complete combustion was expressed as

$$\sigma = 1 + 0.25\beta + \varepsilon - 0.5\gamma. \tag{12}$$

Minimum mole number of dry air required for this process is

$$n_{\mathrm{da_{min}}} = C_{\mathrm{p}}(\sigma + \eta) \tag{13}$$

and minimum required mole number of moist air is

$$n_{\mathrm{ma_{min}}} = C_{\mathrm{p}}(\sigma + \eta + \varphi). \tag{14}$$

In Eq. (14), $\varphi = 7.656 P_{\mathrm{M_{abs}}}\sigma$, $\eta = 3.762\sigma$, and $\sigma = O_{\mathrm{min}}/C_{\mathrm{f}}$. Mole numbers of dry and moist air in the cylinder (n_{da} and n_{ma}) based on excess air coefficient:

$$n_{\mathrm{da}} = \lambda n_{\mathrm{da_{min}}}, \tag{15}$$

$$n_{\mathrm{ma}} = \lambda n_{\mathrm{ma_{min}}}. \tag{16}$$

Air is filled into the cylinder during intake stroke. Nevertheless, the gases cannot be completely emitted from the cylinder following the exhaust stroke, which causes some emission gases to flow back into the cylinder. This leftover gas is due to vacuum pressure as the intake valve opens before the exhaust valve is closed. Therefore, the mole number of air in the cylinder does not solely consist of fresh air filling into the cylinder. Mole number of the overall mixture in the cylinder (n_{mix}) was determined by summing the mole number of inlet air and fuel (n_{fill}) and the mole number of exhaust gases filling back into the cylinder (n_{ex}) as shown in Eq. (17):

$$n_{\mathrm{mix}} = n_{\mathrm{fill}} + n_{\mathrm{ex}}. \tag{17}$$

The mole number of inlet mixture into the cylinder and the mole number of exhaust gas were calculated using Eqs. (18) and (19), respectively. In order to calculate mole number of exhaust gas, exhaust gas temperature (T_{ex}) needs to be calculated first, which is given in Eq. (20):

$$n_{\text{fill}} = \frac{P_{\text{fill}} V_{\text{fill}}}{R T_{\text{fill}}}, \tag{18}$$

$$n_{\text{ex}} = \frac{P_{\text{ex}} V_{\text{c}}}{R T_{\text{ex}}}, \tag{19}$$

$$T_{\text{ex}} = T_{\text{exo}} \left(\frac{P_{\text{ex}}}{P_{\text{exo}}} \right)^{\frac{k-1}{k}}. \tag{20}$$

Temperature (T_{exc}) and volume at exhaust valve closing (V_{exc}) were determined, respectively, with Eqs. (21) and (22):

$$T_{\text{exc}} = T_{\text{ex}} \left(\frac{P_{\text{in}}}{P_{\text{exc}}} \right)^{\frac{k-1}{k}}, \tag{21}$$

$$V_{\text{exc}} = V_{\text{c}} \left(\frac{P_{\text{ex}}}{P_{\text{in}}} \right)^{\frac{1}{k}}. \tag{22}$$

Using Eq. (23), volume of intake/fresh mixture that can be filled into the cylinder was calculated by subtracting the volume at exhaust valve closing from the total cylinder volume:

$$V_{\text{fill}} = V_{\text{c}} - V_{\text{exc}}. \tag{23}$$

Fresh mixture filled into the cylinder at atmospheric conditions heats up as it mixes with leftover gases that could not be emitted from the cylinder. The mixture temperature depends both on temperatures of the fresh mixture and of the exhaust gases in the cylinder. Equation (24) incorporates temperature and mole number of inlet mixture so as to calculate the additional effect of exhaust gas on overall mixture temperature. Therefore, the temperature of the mixture enclosed in the cylinder was calculated by Eq. (25):

$$\Delta T = n_{\text{ex}} \frac{T_{\text{exc}} - T_{\text{mix}}}{n_{\text{ex}} + n_{\text{mix}}}, \tag{24}$$

$$T_{\text{mix}} = T_{\text{fill}} + \Delta T. \tag{25}$$

It was assumed that the mixture consisted of moist air (V_{ma}) and vaporized fuel (V_{vf}) and hence the volume of liquid fuel in the gas mixture was neglected. Mixture volume can be expressed explicitly as given in Eq. (26):

$$V_{\mathrm{mix}} = \frac{n_{\mathrm{ma}} R T_{\mathrm{mix}}}{P_{\mathrm{mix}}} + \frac{n_{\mathrm{f}} R T_{\mathrm{mix}}}{P_{\mathrm{mix}}} V_{\mathrm{vf}}. \tag{26}$$

By assuming all fuel is vaporized in the cylinder the latter equation may be expressed as in Eq. (27) and mole number of fuel n_{f} can be expressed as in Eq. (28):

$$V_{\mathrm{mix}} = \left(\frac{R T_{\mathrm{mix}}}{P_{\mathrm{mix}} n_{\mathrm{f}}} \frac{n_{\mathrm{ma}}}{n_{\mathrm{f}}} \right) + 1, \tag{27}$$

$$n_{\mathrm{f}} = \frac{n_{\mathrm{fill}}}{1 + \dfrac{\lambda O_{\mathrm{min}}}{P_{\mathrm{O_2}}}}. \tag{28}$$

The amount of fuel was calculated by using the mole number of fuel filled into the cylinder in each cycle (n_{f}) and the molecular weight of fuel in kmol:

$$m_{\mathrm{f}} = n_{\mathrm{f}} W_{\mathrm{f}}. \tag{29}$$

The mole number of moist air in the cylinder (n_{ma}) was determined by summing the mole number of refilled gases and the mole number of the fuel:

$$n_{\mathrm{ma}} = n_{\mathrm{ex}} + n_{\mathrm{f}}. \tag{30}$$

Mole numbers of nitrogen and water vapour were calculated respectively by Eqs. (31) and (32) using the percentage of nitrogen in the air and the percentage of water vapour:

$$n_{\mathrm{N_2}} = P_{\mathrm{N_2}} n_{\mathrm{ma}}, \tag{31}$$

$$n_{\mathrm{H_2O}} = P_{\mathrm{H_2O}} n_{\mathrm{ma}}. \tag{32}$$

3.2.2. Equations related to compression, combustion and expansion

In the previous section, an equation was derived to calculate the amount of intake mixture in kmols and is assumed to fill in the cylinder at atmospheric pressure.

Compression occurs as the piston moves between crank dead centre (CDC) and head dead centre (HDC) and initiates with the inlet valve closing. The first law of thermodynamics was used to calculate pressure and temperature at each crankshaft angle during the compression [12]. The first law can be expressed as shown in Eq. (33):

$$\mathrm{d}Q - \mathrm{d}W = \mathrm{d}E. \tag{33}$$

The intake mixture is compressed by the piston from 180° to HDC. The compression continues up to the point of spark advance to be specified. The first law of thermodynamics is expressed in Eq. (34) for this stage:

$$E(T_2) - E(T_1) + \left(\frac{P_1 - P_2}{2}\right)(V_2 - V_1) = \mathrm{d}Q, \tag{34}$$

where ($\mathrm{d}E$) is the internal energy:

$$\mathrm{d}E = E(T + \mathrm{d}T, n) - E(T, n_i). \tag{35}$$

Some simple functions and algorithms were used to find the change in an internal energy as a function of temperature [2]. The enthalpy of mixture (h_i) and specific internal energy of mixture (e_i) were calculated using Eqs. (36) and (37), respectively:

$$h_\mathrm{i}(T) = R\left\{\sum_{j=1}^{5} u_{i,j}T^j\right\}, \tag{36}$$

$$e_\mathrm{i}(T) = R\left\{\left(\sum_{j=1}^{5} u_{i,j}T^j\right) - T\right\}, \tag{37}$$

where $u_{i,j}$ are polynomial coefficients. Specific heat of mixture (C_v) varies as a function of temperature and was calculated using Eq. (38):

$$C_\mathrm{v}(T) = \frac{R}{n_\mathrm{m}}\sum_{i=1}^{N} n_i\left\{\left(\sum_{j=1}^{5} ju_{i,\,j}T^{j-1}\right) - 1\right\}. \tag{38}$$

In the combustion model it was assumed that there were three different zones in the cylinder, namely no combustion zone, pre-combustion zone, and after-combustion zone. It was also assumed that the mixture was homogeneous and all fuel went through combustion process according to the combustion law. The amount of fuel mixture being burned in the cylinder was calculated for each crankshaft angle and the calculated pressure and temperature were taken as the initial values for the successive crankshaft angle. Gas pressure, temperature and combustion products were calculated step-by-step using finite differences. The heat of reaction was added to the equation regarding the first law of thermodynamics as the heat of reaction is liberated during the combustion process:

$$\mathrm{d}Q - \mathrm{d}W = \mathrm{d}E + Q_{\mathrm{rh}} d_{\mathrm{nf}}, \tag{39}$$

where $\mathrm{d}Q$ is heat transmitted to the cooling water [8]. Equation (40) was used to calculate $\mathrm{d}Q$ while heat transmission coefficient was found using Eq. (41):

$$\mathrm{d}Q = A_{\mathrm{ht}} h (T_{\mathrm{int}} - T_{\mathrm{ext}}), \tag{40}$$

$$h = \left(\frac{n_{\mathrm{DS}}}{23.0}\right)\left(1000 - 5000\cos\left(\frac{\theta}{2}\right) - 10\right). \tag{41}$$

Reaction heat (Q_{rh}) was found by using Eq. (42):

$$\begin{aligned} Q_{\mathrm{rh}} = \ & (n(h_{\mathrm{f\_CO_2}} + d_{\mathrm{h\_CO_2}}) + n(h_{\mathrm{f\_H_2O}} + d_{\mathrm{h\_H_2O}}) \\ & + n(h_{\mathrm{f\_N_2}} + d_{\mathrm{f\_N_2}}) + n(h_{\mathrm{f\_O_2}} + d_{\mathrm{f\_O_2}})) \\ & - ((h_{\mathrm{f\_C_8H_{18}}} + d_{\mathrm{h\_C_8H_{18}}}) + O_{\mathrm{min}}\lambda(h_{\mathrm{f\_O_2}} + d_{\mathrm{h\_O_2}}) \\ & + 3.76(h_{\mathrm{f\_N_2}} + d_{\mathrm{h\_N_2}}))\,. \end{aligned} \tag{42}$$

Percentage of fuel burned is a function of crankshaft angle. Amount of fuel that burns at each crankshaft angle (θ) was found by using the ignition timing (θ_{it}) [8]:

$$m_{\mathrm{f}} = 1 - \exp\left\{-6.908\left(\frac{\theta - \theta_{\mathrm{it}}}{\Delta\theta}\right)^4\right\}. \tag{43}$$

The equations and methods given in compression with combustion were used for expansion with combustion while equations in compression without combustion were used for expansion without combustion.

4. Results and discussion

Mathematical expressions given in the previous section were used to perform the cycle analysis of the given LPG engine. A computer code was written to carry out the calculations relevant to air and fuel properties and combustion processes for the cycle analysis. The applicability of the postulated method and the efficiency of the algorithm were tested by comparing the theoretical power that was estimated by using the model to the actual LPG engine power that was provided by the manufacturer.

The model was tested for the excess air coefficient of 1.3, seeking the most appropriate ignition timing to approximate the actual engine power at the maximum torque rating, corresponding to engine speed of 3000 rpm. Temperature, pressure and work graphs were also generated as a function of crank angle at this engine speed. Then, the simulation program was used to determine the power of the engine at 2000, 2500 and 5500 rpm that is the engine speed yielding the maximum power of the engine. The specific results that were obtained are given below.

4.1. Estimated temperature and pressure

Change in temperature depending on crankshaft angle at 3000 rpm is depicted in Fig. 1. The temperature slowly increases as the crankshaft angle moves from 180° to 332° of the crankshaft angle. The slight increase in temperature is expected since compression takes place without combustion before 332°. Temperature rise is sharp after the initiation of combustion and the maximum temperature in the cylinder occurred at 362° with 2258 K degrees.

The maximum pressure in the cylinder was estimated to be approximately 12.1 kPa (Fig. 2). According to gas behaviour and thermodynamics laws, temperature and pressure vary simultaneously in the combustion chamber. Figures 1 and 2 confirm that general shapes of the temperature and pressure curves depict similarities. There exists a similarity in the two graphs: there was gradual pressure increase during compression with combustion, sudden rise during compression with combustion, and sharp decrease after 362°.

The temperature curve shows an irregularity at the beginning of combustion and after all fuel has burned after HDC. These imperfections in temperature and hence in the pressure curves might have been caused by the assumptions of the model, overly simplified combustion equation compared to highly complex nature

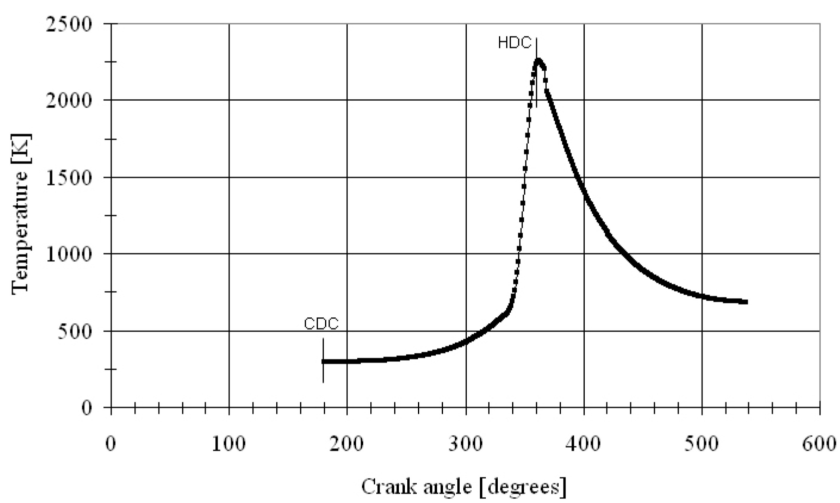

Fig. 1. Temperature change with crankshaft angle.

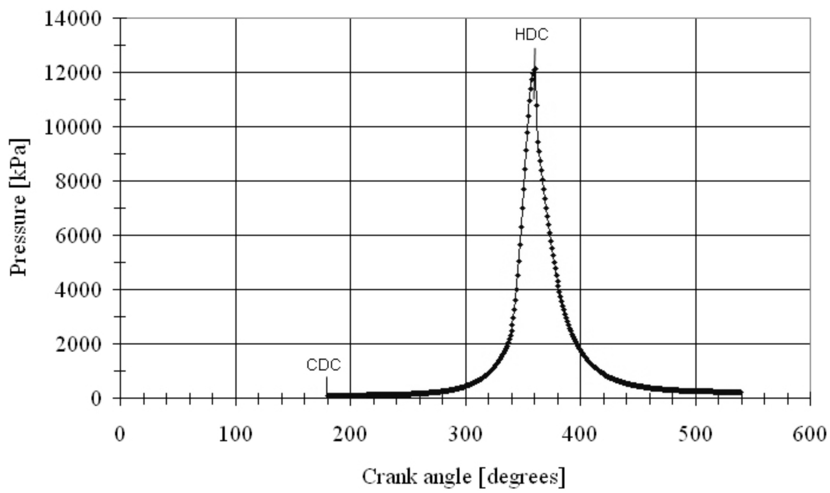

Fig. 2. Pressure change with crank angle at maximum torque rating (3000 rpm).

of combustion processes in the combustion chamber, and errors due to the stopping criteria set in the computer model.

4.2. Work and power estimation

Compression stroke starts at 180° of crankshaft angle and continues until 360°. Work is negative during this period while amount of work given to engine keeps increasing as the degree of crankshaft angle increases (Fig. 3). The work is negative up to HDC and the absolute value of the work is increasing as the crankshaft angle heads HDC. As the cylinder temperature and pressure increases in this period the amount of absolute work increases. The amount of work in each crank angle as shown in Fig. 3 corresponds to cumulative work and hence does not reflect the instantaneous irregularities depicted in temperature and pressure plots.

Indicated engine power was calculated by using the work done per cycle. Power was calculated by multiplying the work at maximum engine power by the number of cycles per second. Power losses are estimated to range from 0.5 to 1.1 kW as the engine speed is varied from 1200 rpm to 3000 rpm. In this study, it was assumed that an average of 0.75 kW power losses occurs for each cylinder at all speeds. As a result, at engine speed of 3000 rpm 35.9 kW was calculated using the model while the true power was 36.3 kW (Fig. 4). Thus, theoretical power had a reasonable agreement with actual power with an error of 1.2 % at this engine speed. Error in other engine speeds, however, showed different deviations from the true engine power and could be as low as 0.39 %. The theoretical power was found to be lower than the true power at engine speeds of 2000 and 5000 rpm, whereas it was larger at speeds of 2500 and 3000 rpm. Consequently, the postulated model accomplished

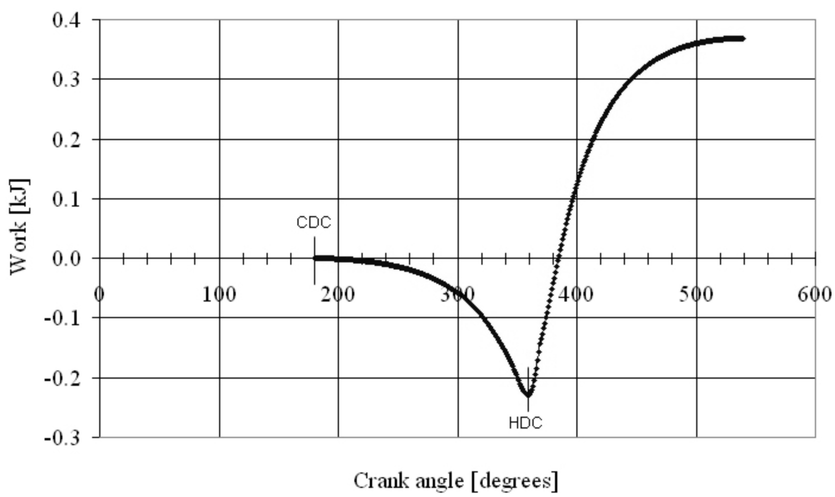

Fig. 3. Work as a function of crank angle at maximum power at 3000 rpm.

a reasonable approximation to the effective engine power with an overall error of 0.72 %.

This study attempted to use the thermodynamic and combustion laws to carry out the engineering calculations relevant to internal combustion engine cycle analysis. The given mathematical models that describe different phases of combustion

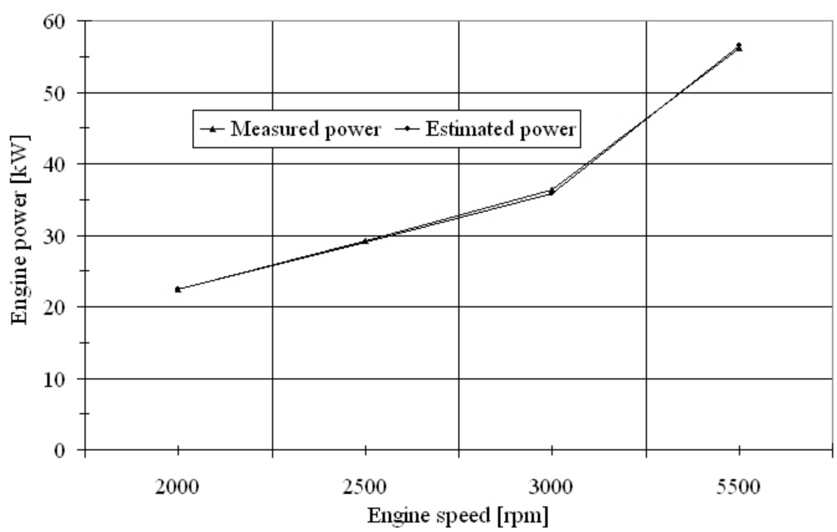

Fig. 4. Theoretical effective power vs. effective engine power at different engine speeds.

process in this study might not exactly reflect what actually takes place in the cylinder. The chemical reaction formulae may be overly simplified considering complex nature of combustion processes and the resulting combustion products. Despite these drawbacks in the model proposed, the results are encouraging in obtaining a useful aid for internal combustion engine cycle analysis.

5. Conslusions

In this study, mathematical equations were set up to do cycle analysis for an engine equipped with an LPG system. A computer code was written to carry out required calculations for fuel and air properties, and cycle analysis. The temperature, pressure, work, and power were determined for the maximum torque rating, i.e. 3000 rpm, while power was estimated also at 2000, 2500 and 5500 rpm to assess the usefulness of the model in determining the performance of the engine. The results of this study could be summarized as follows:

- Maximum temperature in the cylinder was found to be 2258 K degrees at 362° while the maximum pressure in the cylinder was estimated to be approximately 12.1 kPa at 3000 rpm for $\lambda = 1.3$.
- The imperfections in temperature and pressure curves were attributed to the assumptions of the model, the simplified combustion equation compared to complex nature of combustion processes, and potential inefficiencies of the simulation code.
- Theoretical power was estimated at engine speed of 3000 rpm with an error of 1.2 %, implying the need for further development in the combustion model and in the simulation code for more accurate prediction of the true engine power.
- The initial outcome of the study was considered to be promising in developing a useful model that could be an aid in engine performance calculations.

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