Hydrodynamic peculiarities of two-phase vertical bubbly flow

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In the paper, the two-phase vertical bubbly flow in loop bioreactor of airlift type is analysed. Authors start with drift flux model, which adequately characterizes such flow case. Using this theory and a new knowledge about two-phase friction factor, relationships enabling to calculate liquid flux and slip velocity in the reactor riser section are presented. Very good concordance with results of the authors' own measurements and with relationships of another authors was shown also for gas hold-up measurement results. Unknown parameters for this reactor type, which need further detailed research, are summarized.

K e y   w o r d s: two-phase flow, bubbly flow, airlift bioreactor, slip velocity, two--phase friction factor, gas hold-up

Nomenclature

A – area $[\mathrm{m}^2]$

C_w – resistance coefficient $[\mathrm{kg} \cdot \mathrm{m}^{-3} \cdot \mathrm{s}^{-1}]$

d – diameter $[\mathrm{m}]$

d_e – equivalent diameter $[\mathrm{m}]$

F_0 – resistance force $[\mathrm{kg} \cdot \mathrm{m}^{-2} \cdot \mathrm{s}^{-2}]$

g – gravity acceleration $[\mathrm{m} \cdot \mathrm{s}^{-2}]$

H – height $[\mathrm{m}]$

j – volumetric flux (superficial velocity) $[\mathrm{m} \cdot \mathrm{s}^{-1}]$

L – length $[\mathrm{m}]$

n – exponent $[-]$

p – pressure $[\mathrm{Pa}]$

Q – gas feed to bioreactor $[\mathrm{m}^3 \cdot \mathrm{s}^{-1}]$

t – time $[\mathrm{s}]$

V – volume $[\mathrm{m}^3]$

v – velocity $[\mathrm{m} \cdot \mathrm{s}^{-1}]$

v_s – slip (relative) velocity $[\mathrm{m} \cdot \mathrm{s}^{-1}]$

v_Gj – gas drift velocity $[\mathrm{m} \cdot \mathrm{s}^{-1}]$

$\dot{V}$ – volumetric flow rate $[\mathrm{m}^3 \cdot \mathrm{s}^{-1}]$

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Greek symbols

ε_G – gas hold-up [–] τ – shear stress [Pa]
λ – friction factor [–] ∇ – nabla (Hamilton's) operator [m^{-1}]
ρ – density [kg·m^{-3}]

Subscripts

G – refers to the gas   d – refers to the down comer
L – refers to the liquid   r – refers to the riser

1. Introduction

Two-phase vertical bubbly flows are used in various processes of chemical, food and biochemical technologies. They are common at simulation analysis of hydrodynamics, e.g. in airlift loop bioreactors which are schematically illustrated in Fig. 1. Here, a continuum is created by various water solutions in which a gas (mainly air) is distributed into small bubbles. Such flow regime is characterized by its own properties that are different comparing to one-phase flow [1–5].

Nicol and Davidson [6] discovered that momentum eddy diffusivity of the two-phase flow is five to six times higher than in the one-phase flow. This fact must logically influence also friction losses in a flowing medium. Authors [1] published in 1988 their measurement results in ascending two-phase flow water-air bubbles and found that friction factor in such case is not a function of Reynolds number but Froude number and gas (air) hold-up. The relationship is as follows:

$$\lambda = 0.19 \left(\frac{\sqrt{gd}}{j_L} \right)^{1.1} \sqrt{\varepsilon_G}. \tag{1}$$

Kawase [2] used the mentioned notion of Nicol and Davidson about the rise of momentum eddy diffusivity in the two-phase flow and derived theoretically nearly the same equation for the friction factor which is as follows:

$$\lambda = 0.21 \frac{\sqrt{gd}}{j_L} \sqrt{\varepsilon_G}. \tag{2}$$

Equations (1), (2) are valid for the flow in a circular tube. Chriašteľ [3] derived for annulus flow a similar relationship:

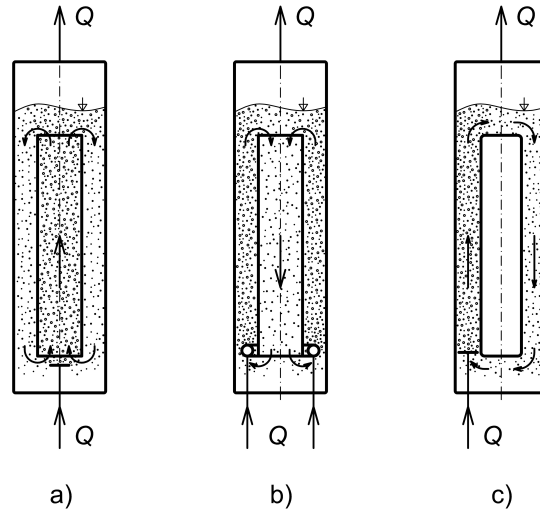$$\lambda = 0.28 \frac{\sqrt{gd_e}}{j_L} \sqrt{\varepsilon_G}. \tag{3}$$

Fig. 1. Basic arrangements of airlift loop bioreactors: a) with internal loop, central tube sparged, b) with internal loop, annulus sparged, c) with external loop.

It is surprising that majority of specialists, working in hydrodynamics, do not respect the mentioned facts about the friction factor in two-phase flow. But, considering the enormous rise of the momentum eddy diffusivity, it is evident that also the friction factor must be higher comparing to the one-phase flow.

Besides the friction factor, we can define also another relevant parameters of the two-phase flow. They are as follows:

X gas hold-up,

X superficial velocity of liquid phase (liquid flux) and gas phase (gas flux),

X slip (relative) velocity between the both phases,

X drift velocity of the gas phase.

2. Drift flux model of two-phase flow

In our considerations, a drift flux model of bubbly two-phase flow will be used. Detailed description and analysis of this flow one can find in excellent Wallis monography [4]. The drift flux model is very useful tool by which the real situation in the loop reactors may be described.

The gas hold-up expresses a gas volume concentration in the gas-liquid mixture and is defined as

$$\varepsilon_{G} = \frac{V_{G}}{V_{G} + V_{L}} = \frac{j_{G}}{v_{G}}. \tag{4}$$

In the drift flux model, the hold-up in ascending coccurent flow is expressed by equation

$$\varepsilon_{\mathrm{G}} = \frac{j_{\mathrm{G}}}{j_{\mathrm{G}} + j_{\mathrm{L}} + v_{\mathrm{Gj}}}, \tag{5}$$

where j_{G} is gas flux

$$j_{\mathrm{G}} = \frac{\dot{V}_{\mathrm{G}}}{A} = \varepsilon_{\mathrm{G}} v_{\mathrm{G}}, \tag{6}$$

j_{L} is liquid flux

$$j_{\mathrm{L}} = \frac{\dot{V}_{\mathrm{L}}}{A} = (1 - \varepsilon_{\mathrm{G}}) v_{\mathrm{L}} \tag{7}$$

and v_{Gj} is gas drift velocity

$$v_{\mathrm{Gj}} = v_{\mathrm{G}} - j = v_{\mathrm{G}} - (j_{\mathrm{G}} + j_{\mathrm{L}}). \tag{8}$$

Wallis [4] proposed the following relationship for the gas drift velocity

$$v_{\mathrm{Gj}} = v_{\infty}(1 - \varepsilon_{\mathrm{G}})^n. \tag{9}$$

Parameter v_{∞} is ascending velocity of single air bubble in the liquid continuum. Exponent n according to Wallis is $n = 2$. This value was precised by Richardson and Zaki, see e. g. [5, 6] onto $n = 2.39$.

For the slip or relative velocity the following relationship is valid

$$v_{\mathrm{s}} = v_{\mathrm{G}} - v_{\mathrm{L}}. \tag{10}$$

The fluxes and actual velocities of the individual phases are in mutual dependences expressed by Eqs. (6) and (7). Starting with Eq. (8), and using Eqs. (6), (7), we can finally obtain a useful relationship between the drift flux and the slip velocity in riser

$$v_{\mathrm{Gjr}} = v_{\mathrm{s}}(1 - \varepsilon_{\mathrm{Gr}}). \tag{11}$$

From Eqs. (9) and (11) the following expression for the slip velocity results

$$v_{\mathrm{sr}} = v_{\infty}(1 - \varepsilon_{\mathrm{Gr}})^{n-1}, \tag{12}$$

see [5, 12]. The slip velocity in the riser can be expressed also as

$$v_{\mathrm{sr}} = \frac{\varepsilon_{\mathrm{Gr}}}{1 - \varepsilon_{\mathrm{Gr}}} j_{\mathrm{Lr}} - j_{\mathrm{Gr}} + v_{\mathrm{Gjr}}, \tag{13}$$

see [5].

The slip velocity can be determined as well by k-ε turbulent model of the two-phase flow [7, 8]. This model is described by the continuity and momentum equations, written for the both phases. Continuity equation for the gas phase is

$$\frac{\partial(\varepsilon_{\mathrm{G}}\rho_{\mathrm{G}})}{\partial t} + \overrightarrow{\nabla} \cdot (\varepsilon_{\mathrm{G}}\rho_{\mathrm{G}}\vec{v}_{\mathrm{G}}) = 0 \tag{14}$$

and for the liquid phase

$$\frac{\partial[(1 - \varepsilon_{\mathrm{G}})\rho_{\mathrm{L}}\vec{v}_{\mathrm{L}}]}{\partial t} + \overrightarrow{\nabla} \cdot (1 - \varepsilon_{\mathrm{G}})\rho_{\mathrm{L}}\vec{v}_{\mathrm{L}} = 0. \tag{15}$$

Momentum equation for the gas phase is

$$0 = -\varepsilon_{\mathrm{G}}\overrightarrow{\nabla}p + \varepsilon_{\mathrm{G}}\rho_{\mathrm{G}} - F_0 \tag{16}$$

and for the liquid phase

$$\begin{aligned}
(1 - \varepsilon_{\mathrm{G}})\rho_{\mathrm{L}}\frac{\partial\vec{v}_{\mathrm{L}}}{\partial t} &+ (1 - \varepsilon_{\mathrm{G}})\rho_{\mathrm{L}}\vec{v}_{\mathrm{L}} \cdot \overrightarrow{\nabla}\vec{v}_{\mathrm{L}} \\
&= (1 - \varepsilon_{\mathrm{G}})\overrightarrow{\nabla} \cdot \vec{\tau}_{\mathrm{L}} - (1 - \varepsilon_{\mathrm{G}})\overrightarrow{\nabla}p + (1 - \varepsilon_{\mathrm{G}})\rho_{\mathrm{L}}\vec{g} + \vec{F}_0.
\end{aligned} \tag{17}$$

Parameter F_0 in Eqs. (16) and (17) is a resistance force, representing an interactive member between the two phases which is equal

$$F_0 = C_{\mathrm{w}}\varepsilon_{\mathrm{Gr}}v_{\mathrm{s}}, \tag{18}$$

whereby $C_{\mathrm{w}} = 5.10^4$ kg $\cdot$ m^{-3} $\cdot$ s^{-1} is a resistance coefficient. By substitution of this value into Eq. (16) and considering equality $\overrightarrow{\nabla}p = \rho_{\mathrm{L}}\vec{g}$ we obtain finally that $v_{\mathrm{s}} = 0.2$ m $\cdot$ s^{-1}. The same value of the slip velocity is obtained also by Eq. (12) when the air bubble diameter is $d_{\mathrm{B}} = 4$ mm, the air hold-up $\varepsilon_{\mathrm{Gr}} = 0.07$ and exponent $n = 2.39$. But Eq. (12) is more complex because it considers a type of liquid, the air bubble diameter and the air hold-up.

All our previous considerations were concentrated onto riser reactor part where ascending coccurent flow occurs. Different situation exists in down comer section; here the liquid moves down but the gas, being drawn by the liquid, tries to flow

in up-direction, due to the Archimedes buoyancy force. Then, respecting the drift flux model, we may write for the gas hold-up in this section that

$$\varepsilon_{\mathrm{Gd}} = \frac{j_{\mathrm{Gd}}}{j_{\mathrm{Gd}} + j_{\mathrm{Ld}} - v_{\mathrm{Gjd}}} \tag{19}$$

and for the slip velocity

$$v_{\mathrm{sd}} = \frac{\varepsilon_{\mathrm{Grd}}}{1 - \varepsilon_{\mathrm{Gd}}} j_{\mathrm{Ld}} - j_{\mathrm{Gd}} + v_{\mathrm{Gjd}}. \tag{20}$$

As for the slip velocity and the friction factor in the down comer, these parameters need detailed research in order to obtain relationships similar to those received for the riser section.

Next task will be to determine local resistance factors of the flow turn in the upper and lower reactor part, respectively. Authors [9, 10] searched this problem but for the one-phase flow only. Some possibility is given in [11] where results of Russian authors are published but also this problem stays open.

2.1 Published relationships for the gas hold-up and liquid flux

Some published relationships respect the drift flux model for the hold-up determination. Declinations are due to the fact that the two-phase flow had not strictly bubbly character at performed tests. Nicol and Davidson [6] proposed the following equation for the airlift reactor with external loop:

$$\varepsilon_{\mathrm{Gr}} = \frac{j_{\mathrm{Gr}}}{1.13(j_{\mathrm{Gr}} + j_{\mathrm{Lr}}) + 0.28}, \tag{21}$$

valid for the riser section, and

$$\varepsilon_{\mathrm{Gd}} = \frac{j_{\mathrm{Gd}}}{j_{\mathrm{Gd}} + j_{\mathrm{Ld}} - 0.21}, \tag{22}$$

valid for the down comer section.

In [11], a relationship of Zuber and Findlay is given which is as follows

$$\varepsilon_{\mathrm{Gr}} = \frac{j_{\mathrm{Gr1}}}{1.6(j_{\mathrm{Gr1}} + j_{\mathrm{Lr}}) + 0.25}, \tag{23}$$

and a relationship of Heijnen, van't Riet in the form

$$\varepsilon_{\mathrm{Gr}} = \frac{j_{\mathrm{Gr1}}}{0.9(gd_{\mathrm{r}} j_{\mathrm{Gr1}}) + 0.25}, \tag{24}$$

where

$$j_{\mathrm{Gr1}} = \frac{Q}{A_{\mathrm{r}}}. \tag{25}$$

Equations (23) and (24) are valid for riser part of the airlift reactor with internal circulation.

The liquid flux influences a quality of all transport phenomena in a bioreactor, and so it contributes to the more effective product and biomass yields. Therefore, there is an effort to rise this parameter, magnitude of which depends on density differences in both bioreactor parts. For that reason, some researchers try to separate air at exit from the riser section. It is true that such an idea rises driving force for the liquid circulation, but it is necessary to take into account also the oxygen utilization degree in the bioreactor. Tests have shown that in conventional airlift bioreactor not more than 25 % of oxygen is transferred from air bubbles into the liquid continuum. Moreover, our measurements proved nearly the same oxygen transfer coefficients for the both reactor sections [13].

In the literature sources, several equations exist for the liquid flux calculation. Majority of them is purely empirical but some respect the theoretical basis. Detailed review of these relationships is possible to find in [5, 12, 14] where their authors proposed a relationship, considering all the known knowledge describing the two-phase flow. The equation reflects steady state in the bioreactor when driving force is equal to the friction losses and drops. It is as follows:

$$\begin{aligned}
&\left[\frac{\left(\dfrac{A_{\mathrm{d}}}{A_{\mathrm{r}}} \right)}{2(1 - \varepsilon_{\mathrm{Gr}})^2} + 1 \right] j_{\mathrm{Ld}}^2 \\
&+ \frac{H\sqrt{g}}{2} \left[0.21(1 - \varepsilon_{\mathrm{Gr}})\sqrt{\frac{\varepsilon_{\mathrm{Gr}}}{d_{\mathrm{r}}}} \frac{A_{\mathrm{d}}}{A_{\mathrm{r}}} + 0.28(1 - \varepsilon_{\mathrm{Gd}})\sqrt{\frac{\varepsilon_{\mathrm{Gd}}}{d_{\mathrm{ed}}}} \right] j_{\mathrm{Ld}} \\
&- Hg(\varepsilon_{\mathrm{Gr}} - \varepsilon_{\mathrm{Gd}}) = 0.
\end{aligned} \tag{26}$$

3. Experimental results and their comparison with the mathematical models

Our measurements were performed in airlift bioreactor with internal loop, outer tube diameter was 150 mm, inner tube diameters 90/84 mm, length 1005 mm. Experimental plant is illustrated in Fig. 2. For the hold-up determination in the both bioreactor parts, a manometric method was used. Received results are in Fig. 3, where their comparison with the Eqs. (23), (24) is also illustrated. It is visible that our values are in very good concordance with Eq. (24) proposed by Heijnen and van't Riet.
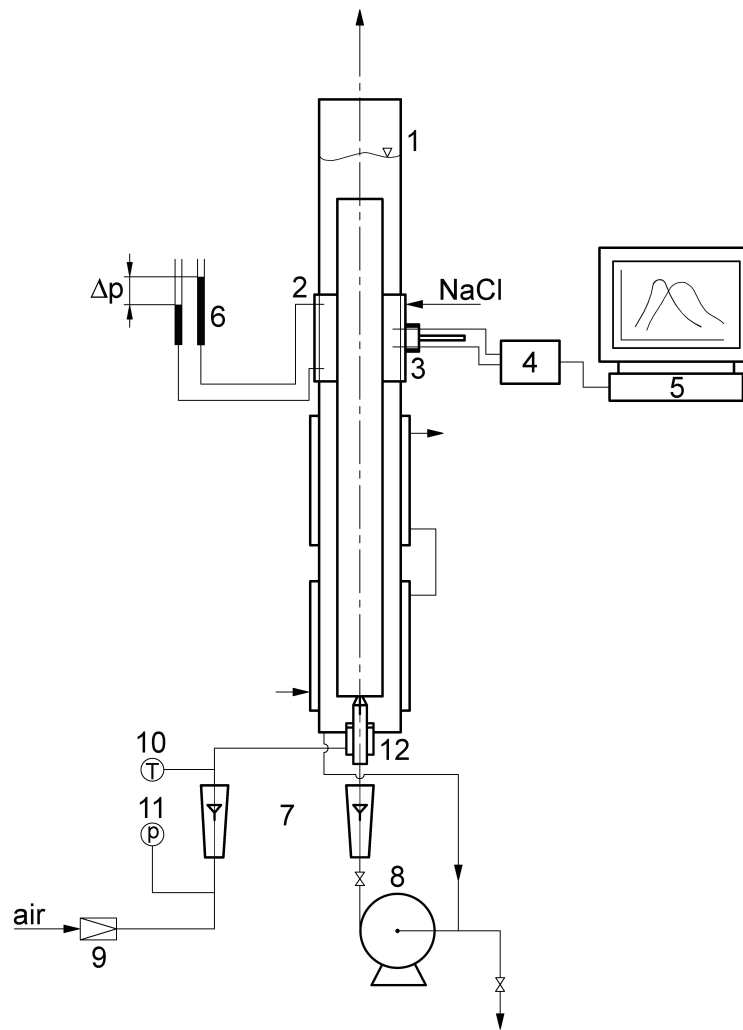
Fig. 2. Scheme of experimental plant which can operate in the following two modes: a) in airlift mode (without the pump), b) in jet loop mode; 1 – reactor body, 2 – gas hold--up measurement, 3 – velocity measurement, 4 – conductometer, 5 – transducer and PC, 6 – U-manometer, 7 – rotameter, 8 – water pump, 9 – pressure reduction, 10 – thermometer, 11 – manometer, 12 – liquid-gas distributor.

   Liquid fluxes were received by conductivity method, described in detail in [3]. Results of our measurements are in Fig. 4 together with curve calculated according to Eq. (26). Both values are in very good concordance. The same fact can be stated also when we compare our results with Chisti's ones, see e.g. [5].
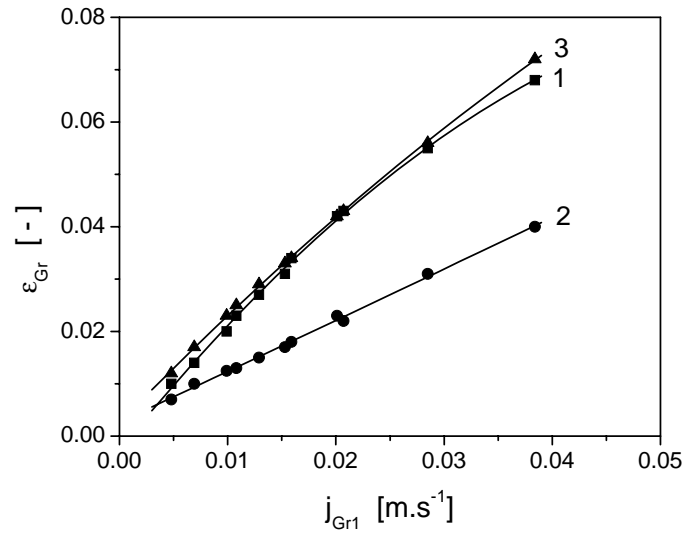
Fig. 3. Gas hold-up in the riser, comparison of results of various authors: 1 – measured values, 2 – values according to Eq. (23), 3 – values according to Eq. (24).
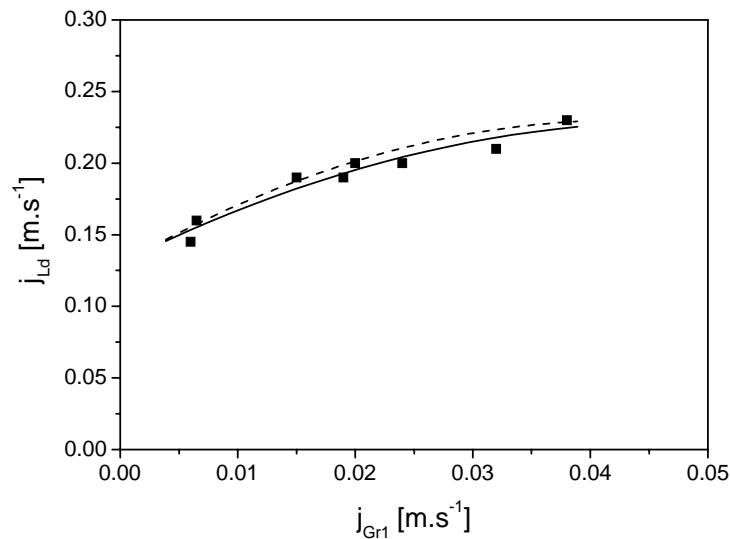

Fig. 4. Comparison of measured and calculated values of liquid flux, points and continuous line represent the measured values, dashed line the values calculated according to Eq. (26).

4. Conclusions

Our tests, relationships and mutual comparison of results, and the comparison with another authors revealed that drift flux model is very suitable for the two-phase vertical bubbly flow description. Despite of this fact, four relevant parameters

of this flow are still unknown and ask for detailed analysis and research. They are as follows:

1. friction factor of the bubbly two-phase flow in the down comer section of a loop bioreactor,

2. local friction factors at flow reverse in the upper and lower reactor parts,

3. drift velocity in the down comer section of a loop bioreactor,

4. slip velocity in the down comer section of a loop bioreactor.

It will be interesting to find out if the Eqs. (1) – (3) will be valid also for the down comer part of the loop bioreactor because here the gas brakes the liquid downstream flow due to the Archimedes buoyancy force. It would be reasonable to assume that in such case the momentum eddy diffusivity will be higher comparing to the ascending coccurent two-phase flow.

These tests are pretentious as for the experimental plant and also for the measurement devices and methods.

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