Allocation of joint backlashes in manipulator linkages with emphasis on platform-type robot

ATLURI CHAKRADHARA RAO[1*], ADUSUMILLI SRINATH[2]

Joint clearances in linkages and backlashes in gear trains lead to "position errors". Accuracy suffers if clearances and backlashes are allocated liberally while tighter allocation leads to higher manufacturing cost. Hence acceptable procedure is to allocate joint clearances or backlashes in such a way that the maximum position error is limited to a specified value. A simple method based on the concepts of parallelism and optimization is proposed. This method is extended to platform-type spatial robots.

Key words: allocation, joint clearances, backlashes, graphs, parallelism, robot

1. Introduction

Joint clearances play an important role in mechanisms and machines. On one hand it is essential to permit relative motion between two components and at the same time it leads to position errors, joint clearances can be eliminated by preloading the pairing elements but preloading may result in high noise, power loss and excess wear. It is important to quantify the effect of joint clearances on the output in order to allocate the joint clearances that allow the mechanism to achieve the expected performance. Allocation of close clearances increases the manufacturing cost. Thus optimum allocation of joint clearances or backlashes is desirable. Some methods are presented [1–14].

However, most of the above publications are mathematically more rigorous and are not convenient for practical application. Hence, simple methods are desirable. In the present work an attempt is made to meet this requirement. Closed kinematics chains with multi d.o.f. (degree-of-freedom) can be considered as manipulator structure for heavy duty application. In these structures, parallelism exists between the links. Noting that the position error is cumulative of clearances

[1] Disha Institute of Management and Technology, Raipur (C. G.), India
[2] Koneru Lakshmaiah College of Engineering, Green fields, Vaddeswaram, Guntur District, Andhra Pradesh, India
[*] Corresponding author, e-mail address: atluri_chakradharrao@yahoo.co.in

of all the joints in series, i.e. all the joints in one path between two links, and also noting that there are at least two paths between any two links, i.e. parallelism, mathematical relations are proposed to get the resultant positional error. Principles of optimization are then applied in order to get the permissible joint clearances so that the position error at the output link does not exceed the specified limit.

2. Method

Positional error is the difference between the expected position of a link and its actual position which will be different due to joint clearances. It is widely accepted that in serial manipulators the joint clearances have cumulative effect, i.e. positional error in the output is equal to the sum of the clearances of all the joints.

$$\text{Positional error at the output} = \sum_{i=1}^{j} C_i, \tag{1}$$

where C_i is the clearance at i-th joint and j is the number of joints in series. The clearance C_i is the distance between the centres of the journal and the bearing.

In case of planar in-parallel manipulators which are closed kinematic chains, parallelism exists between every pair of links.

The extent of parallelism that exists between any two links of a closed kinematic chain can be estimated by using the formula reported in [15]. Parallelism between links implies existence of two or more independent paths consisting of links or joints which are not repeated in any other path. Positional error between two links depends upon the number of joints along independent paths connecting the links, i.e. it depends on the parallelism that exists between the links. Obviously distinct chains will have different positional errors, i.e. they depend upon the configuration of the chain.

For example consider a six-link chain, Fig. 1. Links are numbered 1, 2, 3, etc. Parallelism for instance between, say, links 1 and 3 exists because there are three independent paths between them. These paths are (i) Via link 2, (ii) Via link 4 and (iii) Via links 5 and 6. The parallelism P_{1k} between any two links – 1 and k – can be expressed by the relation

$$P_{1k} = \sum_{i=1}^{n} \frac{1}{J_i}, \tag{2}$$

where J_i is the number of joints in the path i, and n is the number of independent paths. J_i in fact is the kinematic distance along path i and J_i shows the extent of nearness, i.e. smaller the value

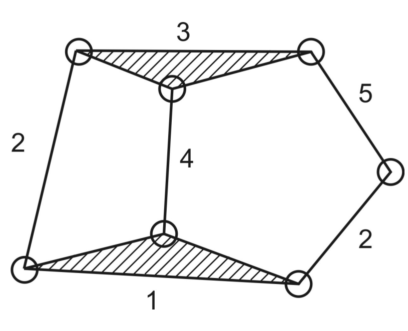

Fig. 1. Six-link chain.

closer are the links or more parallel.

This can lead to same value of parallelism between two links if and if paths are of the same size. This is possible for a pair of links pertaining to two different chains but when it comes to distinct chains as a whole, parallelism of the chains will be different and hence does not lead to any confusion. The above concept can be utilized to get the positional error of one link with respect to the other.

Let C_{1k} be the positional error of link k with respect to link 1 due to clearances of the joints along different independent paths between them. In view of computational convenience it is adequate to consider two shortest paths between the links, i.e. ignoring other paths will not lead to loss of much accuracy. The motion transfer between two links is most influenced by the links along the shortest of the many paths that may exist between them. In general, the influence of the links on motion transfer vanes away as the number of links in a path increase. Hence only shorter paths are considered, now we can express

$$\frac{1}{C_{1k}} = \frac{1}{\sum\limits_{i=0}^{J_1} C_i} + \frac{1}{\sum\limits_{j=1}^{J_2} C_j}, \tag{3}$$

where C_i is the clearance of joint i along path 1 and C_j is the clearance of j-th joint along path 2. J_1 and J_2 are respectively the number of joints along paths 1 and 2.

Equation (3) is suggested on the basis that (i) parallelism will not lead to cumulative error and (ii) as the serialism in one of the paths increases the error justifying the fact that parallelism reduces the error. It is shown in [11] that more parallel chains are less sensitive to joint errors. For illustration consider linkages (a) and (b), Fig. 2.

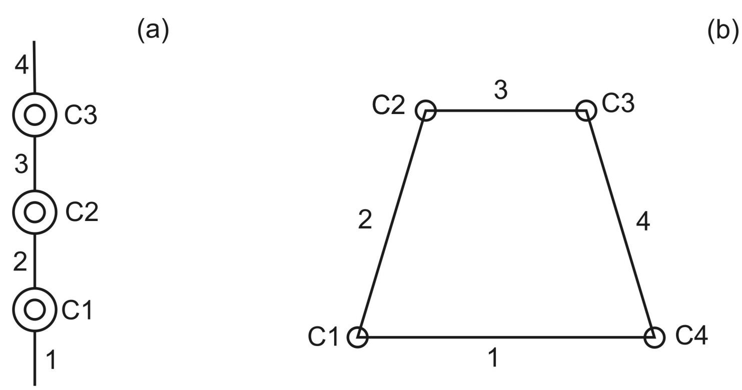

Fig. 2a,b. Four-bar linkages.

For the linkage, Fig. 2a, the positional error of link 4 with respect to link 1 is $C_1 + C_2 + C_3$, where C_1, C_2, C_3 are the clearances of the joints that are in series between the links 1 and 4.

Now consider the four-bar linkage, Fig. 2b. Links are numbered 1, 2, etc., while the clearances C_1 etc. are shown at the joints.

Let us consider links 1 and 2. Though they are directly joined, i.e. one path, there exists another path between them via links 4 and 3, thus links 1 and 2 are parallel.

Parallelism P_{12}, using Eq. (2):

$$P_{12} = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}$$

and the positional error C_{12} is obtained using Eq. (3):

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{(C_4 + C_2 + C_3)}, \tag{4}$$

since the joints 2, 3 and 4 are in series along the second path between links 1 and 2; first path being via joint 1.

From Eq. (4):

$$C_{12} = \frac{C_1(C_2 + C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}. \tag{5}$$

For a given value of the sum $(C_1 + C_2 + C_3 + C_4)$, C_{12} will be maximum, when

$$C_1 = (C_2 + C_3 + C_4). \tag{6}$$

Specifying the maximum permissible position error, say, at joint 1, i.e. C_1, the joint clearances of other joints can be decided to satisfy the relation (6).

If the four-bar chain, Fig. 2b, is used as a path generator, then the position error of link 3 with respect to ground link 1 assumes importance. Positional error C_{13} is expressed as follows:

$$\frac{1}{C_{13}} = \frac{1}{(C_1 + C_2)} + \frac{1}{(C_3 + C_4)},$$

or

$$C_{13} = \frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}.$$

C_{13} will be maximum when $(C_1 + C_2) = (C_3 + C_4)$.

Let

$$(C_1 + C_2) = (C_3 + C_4) = a,$$

then

$$C_{13} = \frac{a \cdot a}{(a + a)} = \frac{a}{2}.$$

Specifying the maximum permissible position error C_{13}, the upper limit of joint clearances can be estimated.

For example, let $C_{13} = 0.0015$ (length dimensions).

$$C_{13} = 0.0015 = \frac{a}{2} \qquad \text{or} \qquad a = 0.003.$$

Hence, $C_1 + C_2 = 0.003$.

Assuming $C_1 = C_2$,

$$C_1 = 0.0015, \qquad C_2 = 0.0015.$$

Similarly, $C_3 = 0.0015$ and $C_4 = 0.0015$.

An important observation is that the positional error between links 1 and 2 will be C_1 if they are connected through one path only.

The effect of the second path via links 3 and 4 is that the positional error C_{12} is reduced, i.e. $C_{12} < C_1$ as one can see from Eq. (5),

$$\frac{C_{12}}{C_1} < 1.$$

For further illustration consider the six-link chain, Fig. 3 same as Fig. 1, but joints are numbered 1, 2, etc. Links are indicated by letters A, B, C.

Positional error

$$\frac{1}{C_{\mathrm{AB}}} = \frac{1}{(C_1 + C_2)} + \frac{1}{(C_3 + C_4)}, \tag{7}$$

where C_1 and C_2 are the clearances at the joints 1 and 2 in series along one of the shortest paths between links A and B.

Similarly, C_3 and C_4 are the clearances of the joints 3 and 4 in series along the second shortest path. There is another path between them consisting of the joints 5, 6 and 7, but the effect is not included to simplify computation and interpretation.

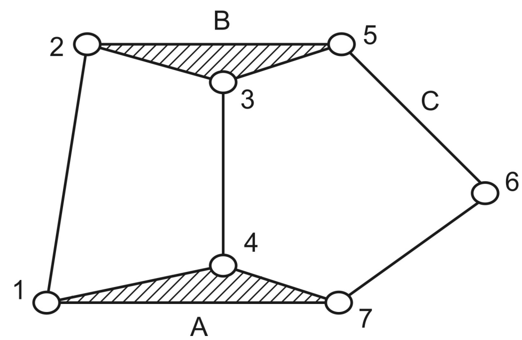

Fig. 3. Six-link chain.

From Eq. (7)

$$C_{\mathrm{AB}} = \frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}. \tag{8}$$

For a specified sum $(C_1 + C_2 + C_3 + C_4)$, maximum positional error between links A and B occurs, when $(C_1 + C_2) = (C_3 + C_4)$.

Specifying the maximum error C_{AB}, one can determine C_1 and C_2 etc.

As another example consider the positional error C_{AC}.

Then

$$\frac{1}{C_{\mathrm{AC}}} = \frac{1}{(C_6 + C_7)} + \frac{1}{(C_3 + C_4 + C_5)}.$$

The above relation gives

$$C_{\mathrm{AC}} = \frac{(C_6 + C_7)(C_3 + C_4 + C_5)}{(C_3 + C_4 + C_5 + C_6 + C_7)}. \tag{9}$$

For a given sum of the denominator, C_{AC} assumes maximum value when

$$(C_3 + C_4 + C_5) = (C_6 + C_7). \tag{10}$$

This relation helps in fixing the joint clearances when maximum permissible error is specified.

Deciding upon backlashes in gear trains is not that straight as in linkages particularly when more carriers (arms) are involved in planetary gear trains. Estimation of position error needs identification of parallel paths but the presence of more carriers and more fundamental circuits makes it a little difficult. It is easy to deal with gear trains in the form of graphs. A graph consists of vertices and

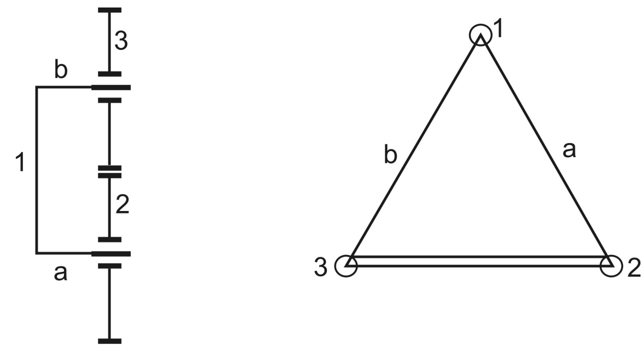

Fig. 4. Schematic diagram of a gear train and its graph.

edges joining the vertices. Every element of a gear train is represented by a vertex in a graph while every joint (pair) is represented by edge (s) or line (s) joining the vertices. In gear trains there are two types of pairs, i.e. (i) Turning pair edge, (ii) Gear pair edge.

Turning pair edge is shown by a single line in the graph while a gear pair is shown by a double line (edge). For example, Fig. 4 shows the schematic diagram of a gear train and its graph. Letters a and b show the level at which the wheels are mounted on the carrier. In Fig. 4, vertex 1 is the carrier, vertices 2 and 3 are the gear wheels that form a gear pair. A turning pair is assigned a numerical weight one since it has one degree of freedom (d.o.f.) while the gear pair is assigned a weight two since it has two d.o.f. Similarly, a spherical pair is assigned a weight of three since it has three d.o.f. For every gear pair there is a fundamental circuit, i.e. it involves the gear wheels and the carrier. In general, when there are more elements in a gear train, there is a fundamental circuit for every pair; the carrier for a gear pair is called a transfer vertex. This can be identified by noting (labelling) pair edges. For a vertex to be a transfer vertex, the turning pair edges on either side should be at different levels. Also, all the edges on a side should be at the same level. Another condition for labelling the edges is that two edges at the same level must intersect at a vertex. For example, Fig. 5 shows two graphs with four vertices corresponding to two distinct gear trains each with two gear pairs.

In the graph on Fig. 5a there are two fundamental circuits, one for each pair, but both the circuits have vertex 1 as their transfer vertex. In other words, all the gear wheels are mounted on the same carrier at different levels. On the other hand, Fig. 5b also has two gear pairs but two carriers. For the fundamental circuit consisting of vertices 1, 2 and 4, vertex 1 is the transfer vertex while vertex 2 is the transfer vertex for the fundamental circuit consisting of vertices 1, 2 and 3.

Parallelism between various elements (vertices) in the gear train graphs should be estimated with respect to each fundamental circuit and in case any parallelism exists between the transfer links it must be included. This is the essential difference

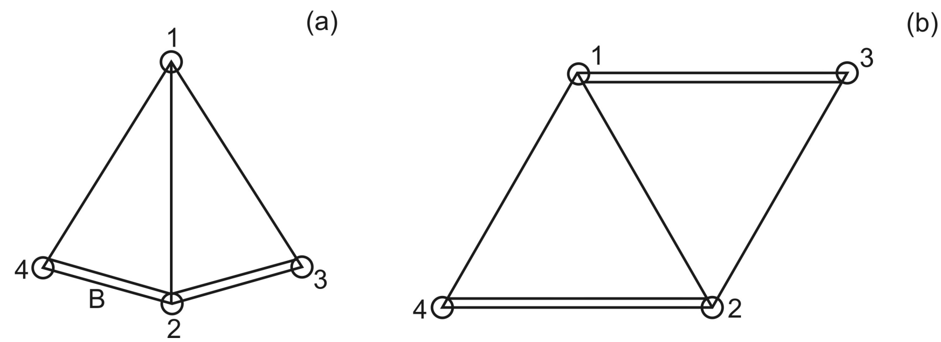

Fig. 5. Two distinct graphs of gear trains with four vertices (a) and two gear pairs (b).

between linkages and gear trains. The gear train – Fig. 5b (graph) has two fundamental circuits (1) and (2). As stated earlier, the position errors can be estimated circuit-wise and the clearances or backlashes can be estimated as illustrated below.

However, there may be instances when an element pertaining to one fundamental circuit is fixed and the output elements (vertex) pertain to a different circuit, e.g. vertices 3 and 4 in Fig. 5b. Parallelism between these two vertices can be estimated by estimating the parallelism of vertices 3 and 4 with respect to their transfer vertices and then including the parallelism between vertices 1 and 2.

Using Eq. (2):

P_{14} – Parallelism of vertex 4 with respect to vertex 1 (since they belong to the same fundamental circuit 1):

$$P_{14} = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}.$$

Similarly, since the vertices 2 and 3 belong to the fundamental circuit 2,

$$P_{23} = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}.$$

Then, since the vertices 3 and 4 belong to different fundamental circuits,

$$\begin{aligned} P_{34} &= P_{14} + P_{23} - P_{12}, \\ P_{34} &= \frac{4}{3} + \frac{4}{3} - \frac{1}{1} = \frac{5}{3}. \end{aligned}$$

Taking clue from the above, we can assign same clearances (C_1) on all the turning pairs and the same backlashes (C_2) on all the gear pairs. With this assumption,

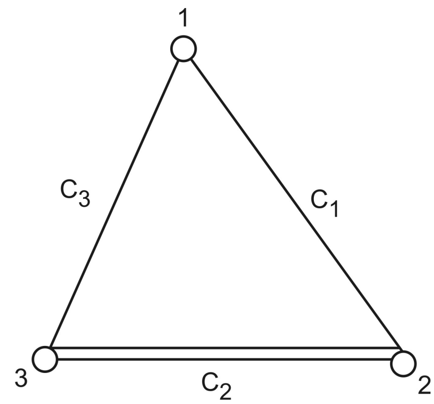

Fig. 6. Joint clearance and backlash of Fig. 4.

for Fig. 5b:

$$\frac{1}{C_{14}} = \frac{1}{C_1} + \frac{1}{(C_1 + C_2)} = \frac{2C_1 + C_2}{C_1(C_1 + C_2)}$$

and

$$\frac{1}{C_{23}} = \frac{2C_1 + C_2}{C_1(C_1 + C_2)}.$$

From the relation for P_{34} explained previously,

$$\frac{1}{C_{34}} = \frac{1}{C_{14}} + \frac{1}{C_{23}} - \frac{1}{C_1} = \frac{3C_1 + C_2}{C_1(C_1 + C_2)}. \tag{11}$$

For illustration consider the gear train on Fig. 4 and its graph. The joint clearances and backlashes are shown in Fig. 6.

The positional errors

$$C_{12} = \frac{C_1(C_2 + C_3)}{(C_1 + C_2 + C_3)}, \tag{11a}$$

$$C_{23} = \frac{C_2(C_1 + C_3)}{(C_1 + C_2 + C_3)} \tag{12}$$

and

$$C_{13} = \frac{C_3(C_1 + C_2)}{(C_1 + C_2 + C_3)}. \tag{13}$$

In this particular case, the maximum permissible position errors C_{12}, C_{13}, C_{23} and the sum of $C_1 + C_2 + C_3 + C_4$ can be specified and the resulting simultaneous equations can be solved to get C_1, C_2 and C_3.

3. Application to platform-type robot

The above approach can be extended to spatial platform-type robots consisting of lower pairs. For example, consider the Fig. 7, in which link 1 is the base and link 2 is the platform; S indicates a spherical pair and R indicates a revolute joint. C_{12}, positional error of the platform with respect to the base:

$$\frac{1}{C_{12}} = \frac{1}{(C_1 + C_2 + 3C_3)} + \frac{1}{(3C_4 + C_5 + C_6)} + \frac{1}{(3C_7 + C_8 + C_9)},$$

where C_1 etc. are the clearances at the joints 1, 2 etc. At spherical joints the error is taken thrice, say, $3C_3$ etc. because of three degrees of freedom.

Obviously, C_{12} will be maximum when

$$(C_1 + C_2 + 3C_3) = (3C_4 + C_5 + C_6) = (3C_7 + C_8 + C_9) = a,$$

$$C_{12} = \frac{a}{3}.$$

Specifying C_{12}, a and hence C_1 and C_2 etc. can be determined.

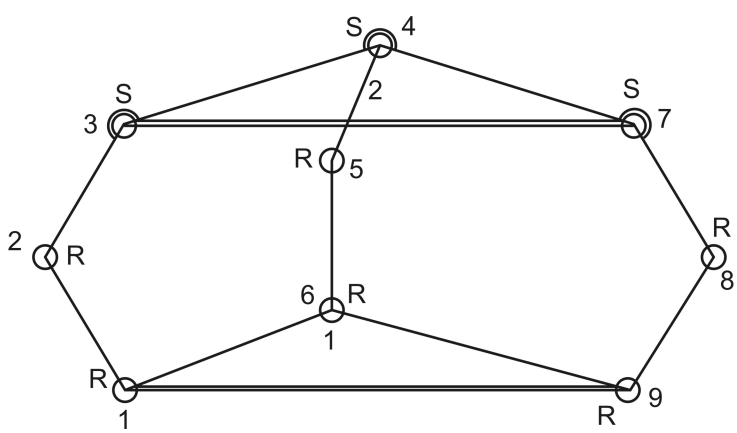

Fig. 7. Linkages of platform-type robots.

4. Conclusion

1. Parallelism reduces the positional error which is always less than the sum of the joint clearances along any of the paths.

2. The given approach leads to set of equations like (8), (10) which will help in fixing/allocating the joint clearances.

3. Maximum positional error occurs when the sum of the joint clearances along each of the parallel paths is the same irrespective of the number of joints in a path.

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