Letter to the Editor

Study of thermoelastic behaviour of skew laminated composite plate with circular cutout using finite element method

A. VENKATA SUDHAKAR[1], V. RAMA CHANDRA RAJU[2],
V. BALA KRISHNA MURTHY[3], K. MOHANA RAO[4],
M. S. R. NIRANJAN KUMAR[1*]

The thermoelastic behaviour of a laminated composite skew plate with hole subjected to non-linear variation of temperature as well as combined temperature and transverse pressure loading has been investigated in the present analysis. The finite element method which works on the basis of three-dimensional theory of elasticity is employed to evaluate the transverse deflection and in-plane stresses. In most of the cases the magnitudes of the transverse deflection and in-plane stresses for combined loading are observed to be less at higher skew angles, because the distance between opposite sides of the plate decreases with increase in skew angle causing an increase in the stiffness of the plate.

Key words: FEM, skew laminate, cutout, interlaminar stresses

1. Introduction

In the design of modern high-speed aircraft and missile structures, swept wing and tail surfaces are extensively employed. Moreover some of the structural elements are provided with cutouts of different shapes to meet the functional requirements like i) for the passage of various cables, ii) for undertaking maintenance work, iii) for fitting auxiliary equipment, etc. Depending upon the nature of application,

---

[1] Production Engineering Department, V. R. S. Engineering College, Vijayawada-520 007, India

[2] R & D Unit, J. N. T. University, Hyderabad-72, Andhra Pradesh, India

[3] P. V. P. Siddhartha Institute of Technology, Vijayawada-520 007, India

[4] C. R. Reddy College of Engineering, Eluru, Andhra Pradesh, India

[*] Corresponding author, e-mail address: m_niranjankumar@rediffmail.com, msr_23nir@yahoo.co.in

these structural elements act upon mechanical and thermal loads of varied nature. Usually, the anisotropy in laminated composite structures causes complicated responses under different loading conditions by creating complex couplings between extensions, bending, and shear deformation modes. To capture the full mechanical behaviour, it must be described by three-dimensional elasticity theories.

In solving the three-dimensional elasticity equations of rectangular plates, quite a number of solution approaches have been proposed. Srinivas and Rao [1] and Srinivas et al. [2] presented a set of complete analytical analyses on bending, buckling and free vibration of plates with both isotropic and orthotropic materials. Setoodeh and Karami [3] employed a three-dimensional elasticity based layer-wise finite element method (FEM) to study the static, free vibration and buckling responses of general laminated thick composite plates. Pagano et al. [4] has given exact solutions for the deflections and stresses of a cross-ply laminated rectangular composites without holes using elasticity theory. Ukadgaonker et al. [5] gave a general solution for bending of symmetric laminates with holes. Morley et al. [6] developed an elementary bending theory for small displacements of initially flat isotropic skew plates without hole. Karami et al. [7] has applied Differential Quadrature Method (DQM) for static, free vibration, and stability analysis of skewed and trapezoidal composite thin plates without hole. From the review of available literature it is observed that the thermoelastic analysis of skew plates with cutouts using elasticity theory has not been studied. The thermoelastic behaviour of a laminate with skew edges and having various types of cutouts is different from the one without skew edges and/or cutouts. So it is necessary to analyse this kind of problem using elasticity theory based on finite element method to evaluate the most accurate behaviour of thick laminated skew plates with cutouts.

2. Problem statement

The present work aims at filling of the knowledge gaps in the existing literature. The research problem deals with the thermoelastic analysis of skew laminated plate with cutouts by elasticity theory based finite element method.

2.1 Problem modelling

2.1.1 Geometric modelling

Figure 1 shows the in-plane of the laminate considered for the present analysis. The dimensions for l and b are taken as 20 mm.

The value of d is determined from the ratio of d/l and the skew angle α is varied from $0°$ to $50°$, the thickness of the plate is fixed from the length to thickness ratio l/h ($s = 10$). The individual layers are arranged so that the total thickness of the layers oriented in x-direction ($\theta = 0°$) is equal to the total thickness of the layers oriented in y-direction ($\theta = 90°$).

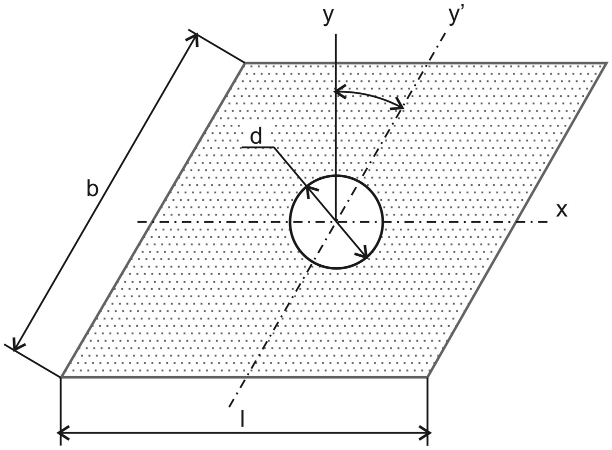

Fig. 1. Skew laminated composite plate with circular cutout.

2.1.2 Finite element modelling

The finite element mesh is generated using a three-dimensional brick elements 'SOLID90' and 'SOLID95' of ANSYS^8 [8]. 'SOLID90' and 'SOLID95' have compatibility to transfer the temperatures from thermal analysis to structural analysis. The 20-node thermal element is applicable to a 3-D, steady-state or transient thermal analysis. The element consists of 20 nodes and temperature degree of freedom for 'SOLID90' and x, y, z directional displacement for 'SOLID95'.

2.1.3 Boundary conditions

i) Thermal:

The temperature of 100 ℃ on the top face and 25 ℃ on the bottom and side faces is applied. The surface of the hole is subjected to convection with film coefficient $h = 5$ and bulk temperature 25 ℃.

ii) Structural:

All the edges of the skew plate are clamped, i.e. all the three degrees of freedom (displacements in global x-, y- and z-directions) of the nodes attached to the side faces of the plate are constrained.

2.1.4 Loading

i) The output from the thermal analysis is applied as thermal loading.

ii) The transverse pressure of 1 MPa is applied on the top surface of the plate in addition to the change in temperature in case of combined loading.

2.1.5 Material properties (graphite-epoxy)

$$K_\mathrm{L} = 36.42\ \mathrm{W/m\cdot K},\qquad K_\mathrm{T} = 0.96\ \mathrm{W/m\cdot K},\qquad E_1 = 172.72\ \mathrm{GPa},$$

$$E_2 = E_3 = 6.909\ \mathrm{GPa},\qquad G_{12} = G_{13} = 3.45\ \mathrm{GPa},\qquad G_{23} = 1.38\ \mathrm{GPa},$$

$$\nu_{12} = \nu_{13} = \nu_{23} = 0.25,\qquad \alpha_1 = 0.57\times10^{-6}/{}^\circ\mathrm{C},\qquad \alpha_2 = \alpha_3 = 35.6\times10^{-6}/{}^\circ\mathrm{C}.$$

3. Validity of the present analysis

To validate the finite element results for structural analysis, a square plate with simply supported edges and subjected to a sinusoidal load of

$$p = p_0 \sin(\pi x/a)\sin(\pi y/b),$$

where a and b are the length and width of the plate, is modelled with 'SOLID95' element. The results obtained from this model are compared with the exact elasticity solution for various lengths to thickness ratios of the plate (Table 1). It is observed that the finite element results are in close agreement with the exact elasticity solution.

In the present work the transverse deflection and in-plane stresses of a clamped skew laminated plate with circular hole at the centre of the plate and subjected to a non-linearly varying temperature across the thickness and transverse pressure are evaluated by varying the size of the hole and skew angle.

Table 1. Comparison of present work with exact elasticity theory

$S = l/h$	Normalized σ_x $(a/2, a/2, \pm1/2)$	Normalized σ_y $(a/2, a/2, \pm1/3)$	Normalized τ_{yz} $(0, a/2, 0)$	Normalized τ_{zx} $(a/2, 0, 0)$	Normalized w $(a/2, a/2, 0)$
10	EL 0.545 −0.545 FE 0.537 −0.536	EL 0.430 −0.432 FE 0.431 −0.431	EL 0.223 FE 0.209	EL 0.258 FE 0.212	EL 0.677 FE 0.692

4. Discussion of results

Numerical results are obtained for two different load cases as mentioned above. Variation of the stresses and deflection with respect to the skew angle (α) and the ratio of diameter of the hole to the side length of the plate (d/l) is shown in Figs. 2–5. The following observations are made:

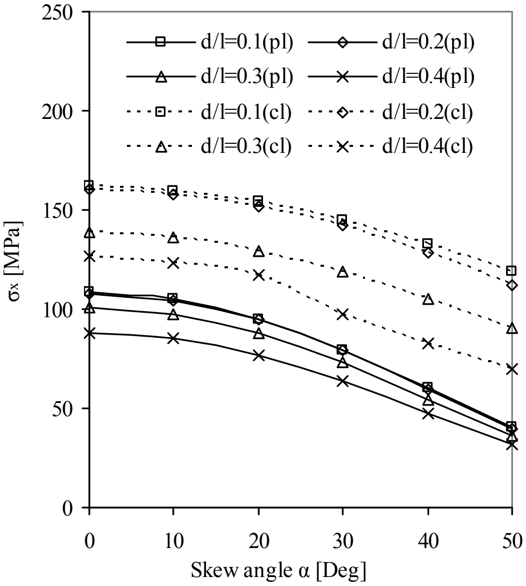

Fig. 2. Variation of σ_x with respect to skew angle for pressure loading (pl) and combined loading (cl).

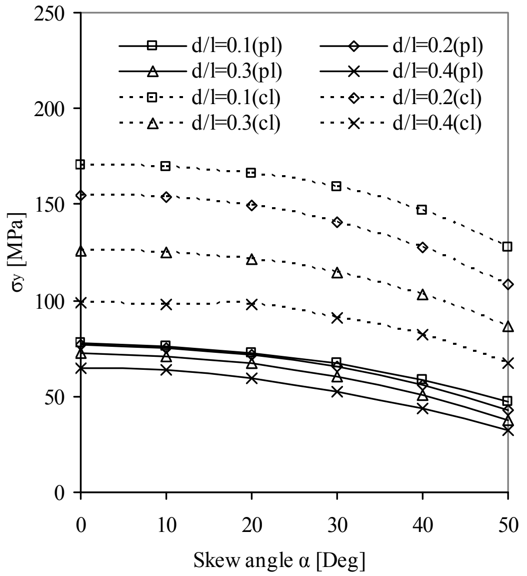

Fig. 3. Variation of σ_y with respect to skew angle for pressure loading (pl) and combined loading (cl).

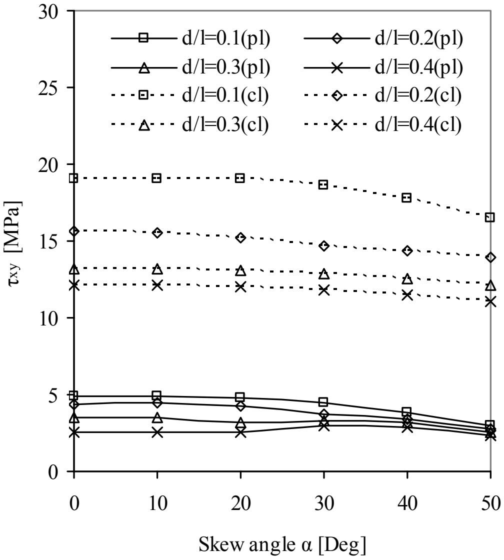

Fig. 4. Variation of τ_{xy} with respect to skew angle for pressure loading (pl) and combined loading (cl).

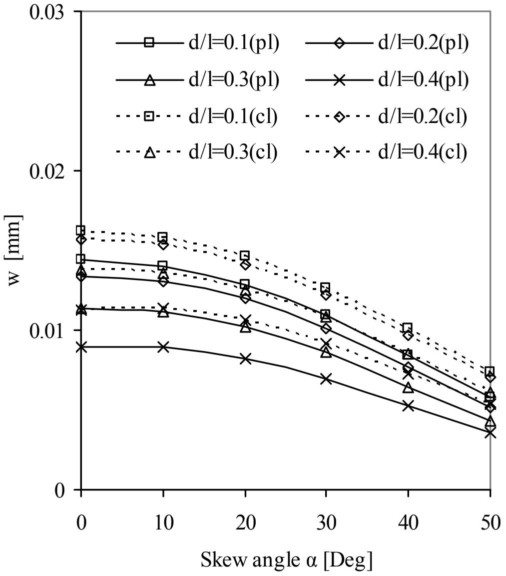

Fig. 5. Variation of w with respect to skew angle for pressure loading (pl) and combined loading (cl).

4.1 Effect of skew angle

When a skew angle is given to a square plate, two of its corners come close to each other whereas other two corners move away from each other. The first effect may cause reduction in stresses and deflection, and the latter effect may cause increase in deflection and stresses. The increase in skew angle increases the length of the longer diagonal and decreases the length of the shorter diagonal of the skew plate. The first factor (increase in the length of the longer diagonal) increases the flexibility of the plate whereas the second factor (decrease in the length of the shorter diagonal) increases the stiffness of the plate. In addition to this the variation of the diameter of the hole influences the stresses and deflection. The resultant effects on the stresses and deflection due to these parameters are explained below.

The normal stress σ_x due to pressure loading (pl) and combined loading (cl) decreases with the increase in skew angle (Fig. 2). The in-plane normal stress σ_y decreases with the increase in skew angle for all the load cases (Fig. 3). In case of pressure loading τ_{xy} increases with increase in α for $d/l = 0.1$ and 0.2 up to $\alpha = 10^\circ$ and then decreases. This stress decreases with increase in α for $d/l = 0.3$ and 0.4. In case of combined loading τ_{xy} decreases with increase in α for all d/l ratios (Fig. 4).

The transverse deflection w decreases with increase in α for pressure loading and combined loading for all d/l ratios. The reduction in w with α may be due to the domination of stiffness factor (Fig. 5).

4.2 Effect of d/l ratio

When the radius of the hole increases, the area supporting the load decreases. Due to this the net force acting on the plate decreases causing for the reduction in stresses. At the same time the resisting volume of the material decreases and as a result the induced stresses will increase. The resultant effect of these factors is discussed below.

The in-plane stresses decrease with increase in d/l ratio for all the load cases. It is observed that the values of the in-plane normal and shear stresses are decreasing due to the increase in d/l ratio, which is due to the reduction in stress concentration with increase in diameter of the hole (Figs. 2–4). The transverse deflection w decreases with increase in d/l ratio for all load cases due to the reason that the resultant force due to the applied pressure decreases with increase in the diameter of the hole (Fig. 5).

5. Conclusions

Thermoelastic analysis of a laminated composite skew plate with a circular hole at the centre of the plate has been carried out in the present work. The transverse deflection and maximum in-plane stresses have been evaluated using

3-dimensional theory of elasticity based on finite element analysis. The results obtained for two different load cases, i.e. pressure loading and combined pressure and non-linearly varying temperature loading, are analysed for the variation of skew angle of the plate and size of the hole. It has been observed that the deflection and stresses are more in case of combined loading when compared to their values for pressure loading. The magnitudes of the in-plane normal stresses and the transverse deflection due to combined loading are greatly affected by the skew angle variation and their magnitudes are observed to be minimum at 50° of the skew angle.

The present analysis helps for the design of safe and efficient structures like skew bridges and swept wings of aircraft structures.

REFERENCES

[1] SRINIVAS, S.—RAO, A. K.: Int. J. Solids Struct., *6*, 1970, p. 1463.
[2] SRINIVAS, S.—RAO, C. V.—RAO, A. K.: J. Sound Vibr., *12*, 1970, p. 187.
[3] SETOODEH, A. R.—KARAMI, G.: Engineering Structures, *26*, 2004, p. 211.
[4] PAGANO, N. J.—HATFIELD, S. J.: AIAA Journal, *10*, 1972, p. 931.
[5] UKADGAONKER, V. G.—RAO, D. K. N.: Comp. Struct., *49*, 2000, p. 41.
[6] MORLEY, L. S. D.: Skew Plates and Structures. Division I: Solid and Structural Mechanics. Int. Series of Monographs on Aeronautics and Astronautics. New York, Pergamon Press 1963.
[7] KARAMI, G.—SHAHPARI, S. A.—MALEKZADEH, P.: Comp. Struct., *59*, 2003, p. 393.
[8] ANSYS Reference manual 2005.

Received: 4.6.2007
Revised: 25.1.2008